



ENGINEERING APPLICATIONS OF NUMBER THEORY

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ABSTRACT

The number theory is a branch of mathematics which is primarily dedicated to the study of integers. The number theory, as such, is less applied in engineering compared to calculus, geometry, etc. The problem was that it could not be used directly in any application. But, the number theory, combined with the computational power of modern computers, gives interesting solutions to real-life problems. It has many uses in various fields such as cryptography, computing, numerical analysis and so on. Here, we focus on the applications of the number theory about engineering challenges.

Key words: Number theory, engineering applications.

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1. INTRODUCTION

Number theory, known as the queen of mathematics is the branch of mathematics that concerns about the positive integers 1, 2, 3, 4, 5 which are often called natural numbers and their appealing properties. From antiquity, these natural numbers classified as odd numbers, even numbers, square numbers, prime numbers, Fibonacci numbers, triangular numbers, etc. Due to the dense of unsolved problems, number theory plays a significant role in mathematics. The recent classification of number theory depending upon the tools used to address the related problems is shown in the Figure 1.

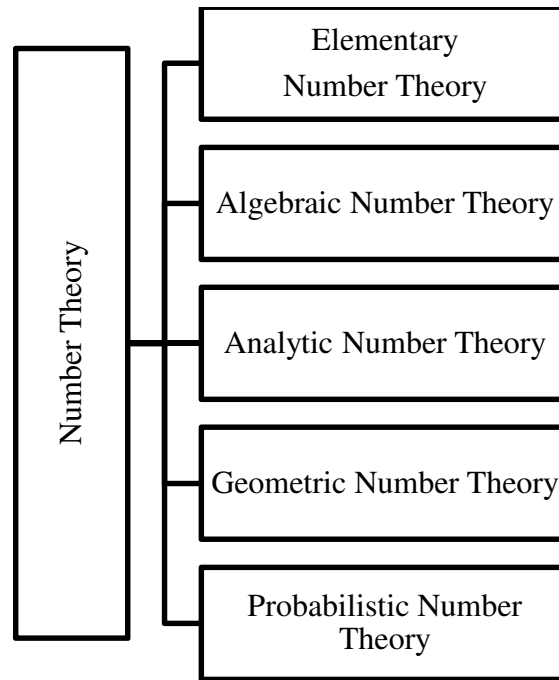


Figure 1. Modern classification on Number Theory

The research on integers in a scientific way is truly credited to Greeks. Later, a big revolution on this theory happened due to the arrival of the famous book “Elements” by Euclid in which the mathematics itself is depicted with precise proofs.

There exist only a few kinds of literature discussing on the applications of number theory in engineering to the best of authors knowledge. So, the objective of present work is to perform a critical review on the existing practices related to the number theory applications in engineering.

2. APPLICATIONS OF NUMBER THEORY

In the early period, number theory, a branch of pure mathematics was practically less applied in real life. But, combined with the present computation technologies, it provides solutions to many current problems. In this section, the authors discussed some of the applications of the theory related to engineering topics.

Cryptography is one of the essential fields in today's digital era, where online security is a big concern. A message sent from a sender to receiver in online communication has the risk of being seen by an unknown person without proper safety. This problem is solved by the use of the concept of encryption/decryption. The message which is sent by the sender is said to be ‘encrypted’ or encoded with the help of a large number, usually prime, which is said to be a ‘key’; the receiver must have that same key to ‘decrypt’ or decode the message. The application of number theory here is in the generation of such large prime numbers. Maurer [1] devised an efficient algorithm to generate such numbers with the help of number theory. Congruence modulo relation is a part of modular arithmetic, a fundamental part of the number theory. The congruence modulo relations, coupled with linear transformations, plays an essential role in cryptography [2]. A natural number expressed as the product of two prime numbers say $a_1 a_2$ where a_1, a_2 need not be distinct is called a semiprime. Semiprimes are exceedingly helpful in the field of cryptography, most distinctly in public key cryptography.

‘Elliptic curves’ is an important concept in number theory. Investigation on number theoretic queries regarding elliptic curves was formerly pursued primarily for inventive reasons. Recently, these queries have become vital in many applied areas, including coding theory, pseudorandom number generation, and chiefly cryptography [4]. There is even unique topic “elliptic curve

cryptography" in the field of cryptology. Coding theory, based on the number theory, serves as an additional shield for the existing cryptic system. Pseudorandom number generation serves the purpose of 'keys' very well. Srikanth [5] discussed the super elliptic Diophantine equation which is an integral part of the field of number theory which is employed for many computer coding based applications.

There are many exciting series of numbers which have utmost importance for problem-solving. One such series is Fibonacci series (0, 1, 1, 2, 3, 5, 8...). It has multiple applications in engineering. In computer science engineering, the 'Fibonacci search technique,' as discussed by Ferguson [6], is a way of searching a sorted array. It uses a divide and conquers algorithm. This algorithm helps to narrow down the possible locations of the required element with the aid of Fibonacci numbers. Fibonacci search splits the array into two segments that have sizes that are successive Fibonacci numbers. It has the convenience that one only needs to perform addition and subtraction to calculate the indices of the accessed array elements instead of other tedious operations. The time dependence of moments and size distributions during consolidation is the utilization of Fibonacci series in the simulation.

Another significant concept related to the Fibonacci series is the golden ratio ϕ (phi). Any two quantities are said to be in the golden ratio if their ratio is equal to the ratio of their sum to the bigger of the two quantities. Represented algebraically, for two quantities x and y , $x > y > 0$, $(x + y)/x = x/y = \phi$. Shapes of several natural and human-made objects are seen to obey the golden ratio [7]. The spirals in the flowers of plants and Parthenon, the famous monument are some of the classic examples. Fibonacci series has found many uses in architecture as well as engineering and is widely seen in nature. The phi code explains the behaviour of structural elements used in engineering. It is seen as a defining parameter in the stress analysis of beams. Collins and Brebbia [8] pointed out the existence of phi code in the relation between normal and shear stresses. The normal stress σ_x and the maximum shear stress τ_{\max} , for the condition $\sigma_x = \tau_{xy}$ and $\sigma_y = 0$, is related as $\tau_{\max} = \sigma_x [\sqrt{5} / 2]$, where $\sqrt{5} = (1 + \Phi^2)/\Phi$. It is a useful tool for structural analysis.

One of the famous theorems in Mathematics is the 'Pythagoras theorem.' It deals with the right-angled triangles, giving the relation between the sides. Unsurprisingly, it has applications in any field which deals with triangles. Few famous examples are listed next. The wing configuration used in modern jet aircraft is the 'Delta wing.' The theorem plays a part in the effective and efficient design of such configuration. Similar applications can be found in tips of rockets, which is an isosceles triangle in sectional view. Sectional analysis of frustum of cones, which serves as fairing between the stages of a multi-stage rocket, is also an example. Calculations of propeller and engine blade angles involve the theorem. Aerospace scientists and meteorologists find a range and sound source using this theorem. An interesting combination of non-arithmetic sequence and the Pythagoras theorem exists in the number theory. The sequence 3, 5, 9, 11, 15, 19, 21, 25, 29, 35, consists of legs as odd numbers in right triangles with the length of the sides being integers and hypotenuse length as a prime number [9].

The acoustic quality of concert halls can be improved with the help of number theory as discussed by Manfred [10]. The construction of new musical scales to the optimum diffusion of sound in the halls improves the acoustic quality to a great practical extent. Methods for improved sound dissipation by reflection phase-gratings, based on three distinct concepts of number theory, are depicted by the work done by Manfred. Boris and Leonid [11] discussed how the restricted partition function could be employed in computing all algebraically independent invariants of the degrees emerging from the action of the finite group on the vector space over the complex field. The use of restricted partition functions to the task of computing "algebraically independent invariants" of the degrees which emerge due to an action of "the finite group on the vector space over the field of complex numbers". A two-parameter generalization of the entire elliptic integral

of the second kind, which is given regarding the Appell function, was discussed by Victor Barsan [12]. This function is further reduced to a quite more comfortable bilinear form in the complete elliptic integrals, and a few real uses are shortly specified related to solid-state physics in this work.

New polynomial equivalents of Jacobi's triple product were given by Krishnaswami Alladi and Alexander Berkovich [13]. A simple introduction to both the mathematical and engineering forms of coding theory, weights of the codewords, were discussed by Robert and Howard [14]. Roger [15] presented actual characteristics of regular point lattices rising from a utilization oriented perspective. He briefly revealed the characteristic of Farey sequences in plant biology. Equivalents of "Newton–Girard power-sum" formulae for whole and meromorphic functions with uses to the Riemann zeta function were discussed by Armen et al. [16].

Finally, an application of Ramanujan sum in engineering is discussed. The form of this sum in signal processing was noticed over the past decades. Vaidyanathan [17] showed how the Ramanujan sum could implement to pull out periodic components in discrete time signals. Again, Vaidyanathan [18] introduced a subspace called Ramanujan subspace and studied its properties for proving the decomposition of finite duration signals into the finite sum of orthogonal subspaces.

Thus the extensive applications of number theory are noted in several areas. The current scenario is such that the role of number theory has more weight for cyber security problems. Future applications are abundant because of the developments in high-speed computers, and there is a scope for the growth in the applications of number theory.

3. CONCLUSION

Various engineering applications of the number theory were mentioned in detail. The significant contribution of number theory in recent years is in the area of cryptography, and hence computer science engineering was noted initially. The importance of famous series and sequences in almost every field of engineering was observed. It is seen that applications of number theory were not directly in some applications; with the number theory being fundamental, it acted as the driving force in approaching the solution. The versatility of the applications was also recognized. Further research and development of the theory will pave the way for more uses of number theory to both pure as well as applied/engineering mathematics.

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